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Gastvortrag

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Global existence and convergence of the Möbius-invariant Willmore flow in the 3-sphere

Abstract:

In this talk, the author aims to present his investigation of the “Möbius invariant Willmore flow” (MIWF), which solves 4th order Willmore-type evolution equation

$$\partial_t f_t = -\frac{1}{2}\frac{1}{\left|A^0_{f_t}\right|^4}\left(\Delta^{\perp}_{f_t} H_{f_t} + \left|A^0_{f_t}\right|^2 H_{f_t}\right) \equiv -\frac{1}{\left|A^0_{f_t}\right|^4}\delta\mathcal{W}(f_t) \qquad (1)$$

on $\Sigma \times [0, T_{\mathrm{Max}})$. Here, Σ denotes an arbitrarily fixed smooth compact torus, $\mathcal{W}$ the Willmore-functional:

$$\mathcal{W}(f) := \int_\Sigma 1 + \frac{1}{4}\left|H_f\right|^2 d\mu_f$$

H_f the mean curvature vector and A^0_f the tracefree part of the second fundamental form A_f of some arbitrary C^2-immersion $f\colon \Sigma \longrightarrow \mathbb{S}^3$:

$$A^0_f := A_f - \frac{1}{2}\, g_f H_f,$$

and $g_f := f^*(\langle\cdot,\cdot\rangle_{\mathrm{euc}})$ denotes the pull-back metric onto Σ of the Euclidean metric on $\mathbb{S}^3 \subset \mathbb{R}^4$. Schätzle proved in 2004 that the classical Willmore-flow for immersions $f_t\colon \mathbb{S}^2 \subset \mathbb{R}^3$ exists for all times $t \in [0,\infty)$ and converges in every C^k-norm to a round sphere, as $t \to \infty$ starting in time $t = 0$ in an arbitrary immersion $F_0 : \mathbb{S}^2 \longrightarrow \mathbb{R}^3$ which satisfies $\mathcal{W}(F_0) < 8\pi$. In 2016, the speaker has already proved existence and uniqueness of smooth short-time solutions to equation (1), starting in arbitrary C^∞-smooth umbilic-free initial immersions F_0 of a fixed compact torus Σ into $\mathbb{S}^n$ (or $\mathbb{R}^n$). Only recently, the speaker proved that this flow exists eternally, in particular does not develop singularities, and subconverges to smooth Willmore-Hopf-tori, if the initial immersion F_0 maps the fixed compact torus Σ onto a “Hopf-torus” in $\mathbb{S}^3$. A Hopf-torus is the preimage $\pi^{-1}(\mathrm{trace}(\gamma))$ of some smooth closed curve $\gamma\colon \mathbb{S}^1 \to \mathbb{S}^2$ .w.r.t. the Hopf-fibration

$$\mathbb{S}^1 \hookrightarrow \mathbb{S}^3 \to \mathbb{S}^2$$

The key-strategy of my proof consists of a “logic descent” of the flow (1) to the flow

$$\partial_t \gamma_t = -\frac{1}{\left(\kappa^2_{\gamma t}+1\right)^2}\left(2\left(\nabla^{\perp}_{\gamma'_t}\right)^2\left(\vec{\kappa}_{\gamma t}\right) + \left|\vec{\kappa}_{\gamma t}\right|^2\vec{\kappa}_{\gamma t} + \vec{\kappa}_{\gamma t}\right) \equiv -\frac{1}{\left(\kappa^2_{\gamma t}+1\right)^2}\delta\mathcal{E}(\gamma t)$$

for closed regular curves $\gamma_t\colon \mathbb{S}^1 \to \mathbb{S}^2$, where

$$\mathcal{E}(\gamma) := \int_{\mathbb{S}^1} 1 + \left|\vec{\kappa}_\gamma\right|^2 d\mu_\gamma$$

denotes the one-dimensional Willmore energy and $\vec{\kappa}_\gamma$ the curvature vector of smooth closed curves $\gamma : \mathbb{S}^1 \to \mathbb{S}^2$.

Eingeladen von Simon Blatt