UNIVERSITÄT SALZBURG

Fachbereich Mathematik
Hellbrunnerstraße 34
5020 Salzburg

Mathematisches Kolloquium

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14.15 Uhr
Seminarraum II

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The Markov kernel perspective of two-dimensional copulas and some useful and surprising consequences

Abstract

Using the one-to-one correspondence between two-dimensional copulas and Markov kernels having the Lebesgue measure on [0,1] as fixed point allows not only for a translation of various well-known copula-related concepts to the Markov kernel setting but also opens the door both to the definition of strong metrics and their induced dependence measures and to surprising mathematical aspects of copulas.

The talk first gathers some basic mathematical properties of copulas and doubly stochastic measures, recalls Sklar's famous theorem, and outlines the interrelation of copulas with Markov operators and Markov kernels. Based on the latter a strong metric D on the space of copulas is defined and some of its main topological properties are stated. More importantly, it is shown how this metric can be used to define a dependence measure with the seemingly natural property of assigning maximum dependence to (and only to) the class of completely dependent copulas – a property that, for instance, Schweizer and Wolff's famous σ does not fulfill. Considering that empirical copulas En are 'almost' completely dependent, in general we cannot expect convergence of En to the underlying copula A with respect to the metric D. Working with empirical Bernstein- or checkerboard copulas, however, helps to overcome this problem and yields strongly consistent estimators with respect to D.

As second major point the well-known star product of copulas will be introduced and the construction of singular copulas (with possibly fractal support) via Iterated Function Systems (IFS) is sketched. Expressing the star product as standard composition of Markov kernels establishes a short and elegant proof of the fact that all idempotent copulas are necessarily symmetric and, additionally, proves helpful when showing that for each s in [1,2] we can find an idempotent copula

As such that the Hausdorff dimension of the support of As is s. Finally, the IFS construction is utilized to show that although copulas are mathematically 'nice' objects (Lipschitz continuous, uniform marginals, etc.) they may exhibit surprisingly singular behaviour – in fact, using tools from

Symbolic Dynamics and the Markov kernel approach once more, we can even construct singular copulas with full support for which all conditional distribution functions are continuous, strictly increasing, and have derivative zero almost everywhere.

Die Organisatoren: Clemens Fuchs, Peter Hellekalek, Reinhard Wolf
http://fuchsc.sbg.ac.at/smc.html