Understanding Non-Categorical Science

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Categoricity arguments, i.e., arguments showing that any two models of an axiomatic theory are isomorphic, have been extensively discussed in contemporary philosophy, under the assumption that the existence of non-isomorphic models divests our belief in the objective and intelligible character of a theory of any justification. This assumption is based on the thought that non-categoricity entails indeterminacy of reference or, even worse, indeterminacy of truth conditions, which is taken to be an insurmountable obstacle to understanding and objectivity. By contrast, early in the twentieth century, mathematicians like Zermelo suggested that non-categoricity should not be considered a defect, but rather an advantage of an axiomatic theory: it increases the generality, the range of applicability, and supports the further development of the theory. Indeed, as an example, non-standard analysis could not have been developed, had the non-categoricity of first-order real analysis been considered a defect.

The present project proposes an investigation of the epistemological and metaphysical virtues of non-categorical theories in mathematics, in fundamental physics, and in logic. This investigation includes three case studies whereby it aims at uncovering novel and rather unexpected aspects of logical validity, mathematical understanding, and physical objectivity. It attempts to recover and build on insights offered by Zermelo and others, while providing solid grounds against the claim (still widespread today among philosophers and logicians) that non-categoricity is a defect, rather than an advantage of axiomatic theories.

The consideration of non-categorical logical systems leads to a generalization of the notion of logical validity in a way that helps reject known counterexamples to modus ponens, and then introduces a correction of the view that logical consequence can be grounded in the formal structure of physical reality. Furthermore, the research clarifies the alleged connection between algebraic closure and mathematical understanding, and develops a new approach to understanding by justifying the explanatory power of proofs in non-categorical mathematical theories. Finally, the research describes the modal implications of non-categorical physical theories like algebraic quantum field theory, and explores a possibilist structuralist view of fundamental physics, i.e., one that conceives of modality in terms of unreduced possible worlds. The project is breaking new grounds by emphasizing and clarifying the role of non-categoricity in achieving these results.

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