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Denial and Disagreement

Julien Murzi · Massimiliano Carrara

Abstract We cast doubts on the suggestion, recently made by Graham Priest, that glut theorists may express disagreement with the assertion of A by *denying* A. We show that, if denial is to serve as a means to express disagreement, it must be exclusive, in the sense of being correct only if what is denied is *false only*. Hence, it can't be expressed in the glut theorist's language, essentially for the same reasons why Boolean negation can't be expressed in such a language either. We then turn to an alternative proposal, recently defended by Beall (in Analysis 73(3):438–445, 2013; Rev Symb Log, 2014), for expressing truth and falsity *only*, and hence disagreement. According to this, the exclusive semantic status of A, that A is either true or false only, can be conveyed by adding to one's theory a *shrieking* rule of the form $A \wedge \neg A \vdash \bot$, where $\bot$ entails triviality. We argue, however, that the proposal doesn't work either. The upshot is that glut theorists face a dilemma: they can either express denial, or disagreement, but not both. Along the way, we offer a *bilateral logic of exclusive denial* for glut theorists—an extension of the logic commonly called LP.

Keywords Disagreement · Dialetheism · Denial · Shrieking · Revenge

Suppose Marc asserts:

(1) Dover is North of London,

and Lisa disagrees.[1] Classically, Lisa may express disagreement by asserting the negation of what Marc said:

(2) Dover is *not* North of London.

Glut theorists, however, may not follow suit: if A is a *glut*, i.e. if it is both true and false, they won't in general take assertions of both A and $\neg A$ to express disagreement.[2] So how, if at all, can they express disagreement? In a number of publications, Graham Priest has suggested that they may do so by *denying* what was said (Priest 1999, 2006a, b). In order for this to work, asserting $\neg A$ must not commit one to denying A, i.e. denial must not be reducible to the assertion of $\neg A$ (Parsons 1984). Thus, glut theorists must reject, and reject, the right-to-left direction of the classical theory of denial, that to deny A is equivalent to asserting $\neg A$:

Classical denial. A is correctly denied iff $\neg A$ is correctly asserted.[3]

The paraconsistent denial of A is *stronger* than the assertion of $\neg A$. Unlike paraconsistent negation, which allows for overlap between truth and falsity (Asenjo 1966;

J. Murzi (✉)
School of European Culture and Language, University of Kent, Cornwallis North West, CT2 7NF Canterbury, Kent, UK
e-mail: j.murzi@gmail.com

J. Murzi
Munich Center for Mathematical Philosophy, Ludwig-Maximilians Universität, Ludwigstrasse 31, 80539 Munich, Germany

M. Carrara
Section of Philosophy, FISPPA Department, University of Padua, Piazza Capitaniato 3, 35139 Padua, Italy
e-mail: massimiliano.carrara@unipd.it

[1] We assume, here and throughout, that the disagreement in question isn't of the faultless kind: at most *one* between Marc and Lisa can be correct as to whether Dover is North of London.

[2] As is well known, the logic of non-trivial theories containing gluts must be paraconsistent, i.e. it may not validate the principle of explosion $A \wedge \neg A \vdash B$. More on this in Sect. 1.

[3] Ripley (2014) calls this the *denial equivalence*.

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Priest 1979, 2006b), denial is assumed to be *exclusive*: assertion and denial are mutually incompatible speech acts (Priest 1993, 1998, 2006a, b).[4]

In this paper, we provide a logic of exclusive denial for glut theorists: a bilateral extension of the logic commonly called LP.[5] We observe that *exclusive deniability*, a key semantic notion of the logic, is not expressible in the glut theorist's language, essentially for the same reasons why Boolean negation is not expressible in such a language either. This is shown by what we call the *Paradox of Deniability*, in effect, a deniability-variant of the Liar Paradox. We then turn to a different proposal, recently defended by Beall, for expressing truth and falsity only, and hence disagreement. According to this, the fact that A has exclusive semantic status—that is either true or false, but not both—can be conveyed by adding to one's system a *shriek* rule of the form $A \wedge \neg A \vdash \bot$, where $\bot$ entails triviality (Beall 2013, 2014). On these assumptions, if $\neg A$ is true, then either A is false only, or one's system is trivial, exactly as in the classical case. We argue, however, that Beall's shrieking proposal does not work either: the glut theorist is not in general in a position to *justify* the addition of shriek rules to a given set of sentences rather than another. We conclude that glut theorists face a dilemma: they can either express denial, or disagreement, but not both.[6]

The paper is structured as follows. Section 1 provides some background. Section 2 introduces LP*, a bilateralist logic for exclusive denial. Section 3 argues that, on natural assumptions, LP* faces well-known, but worth rehearsing, expressive limitations.

[4] For general background on denial in non-classical theories, see Ripley (2011a, Sect. 3).

[5] For general background on LP, see Asenjo (1966), Asenjo and Tamburino (1975), Priest (1979), Routley (1979), Beall (2009).

[6] Our arguments apply, *mutatis mutandis*, to *paracomplete* theories whose logic at least includes the dual of LP, viz. the Strong Kleene logic K3 (Kripke 1975; Field 2008). Just like paraconsistent theories postulate (at least) the possibilities of an overlap between truth and falsity, and thus reject $A \wedge \neg A \vdash B$, paracomplete theories postulate a gap between truth and falsity, and reject the Law of Excluded Middle $B \vdash A \vee \neg A$. Accordingly, paracomplete logicians reject the left-to-right direction of *Classical denial*: in a paraconsistent framework, the denial of A is *weaker* than the assertion of $\neg A$ (see Ripley 2014). This raises the problem of expressing disagreement over a 'gappy' A. For suppose (1) and (2) are 'gappy'. Then, by denying (2) Lisa does *not* express disagreement with Marc's denial of (2), for both denials can be correct. This raises the question how disagreement, and rejection, can be expressed in a paracomplete setting. For reasons of space, we exclusively focus on the paraconsistent case, although we briefly comment on paracompleteness in the parenthetical remark of Sect. 3.1 below.

1 Glut Theories...

Our target theories are glut theories—theories which postulate the existence of gluts on the face of semantical, and perhaps soritical, paradoxes (see e.g. Priest 2006b, a, 2010b; Beall 2009; Colyvan 2009; Weber 2010). To fix ideas, we let the language of these theories include (at least) the language of Arithmetic, supplemented with a monadic predicate $Tr(x)$, expressing truth. We assume that the truth predicate at least satisfies the naïve truth-rules

$$\textit{Tr}\text{-I}\ \frac{\Gamma \vdash A}{\Gamma \vdash Tr(\ulcorner A \urcorner)} \qquad \textit{Tr}\text{-E}\ \frac{\Gamma \vdash Tr(\ulcorner A \urcorner)}{\Gamma \vdash A},$$

where $\ulcorner A \urcorner$ is a (structural-descriptive) name of A. If, as in Priest (2006a) and Beall (2009), a detachable conditional $\to$ is added to the language, we assume that the truth-predicate further satisfies the T-Scheme:[7]

$$(\textit{T-Scheme})\ Tr(\ulcorner A \urcorner) \leftrightarrow A.$$

We take the logic (of the $\to$-free fragment) to be LP.

Semantically, LP is a three-valued logic whose language is that of classical logic (CL). The set of admissible valuations V_{LP} is composed of all the total maps from the set of well-formed formulae *WFF* to the set $\{1, 0.5, 0\}$ satisfying:

($\vee$) $v(A \vee B) = \max\{v(A), v(B)\}$;
($\neg$) $v(\neg A) = 1 - v(A)$;
($\exists$) $v(\exists x\, \Phi) = \max\{v(\alpha(o/x))\}$: for some o in the domain.[8]

The designated values are $\{1, 0.5\}$: an argument is LP-valid if it never gets you from either 1 or 0.5 to 0. When moving to our target theories, we must further ensure, if the truth-predicate is to be *transparent* (i.e. if $Tr(\ulcorner A \urcorner)$ is to be fully intersubstitutable with A in all non-opaque contexts), then the LP-valuations satisfy:

(Tr) $v(Tr(\ulcorner A \urcorner)) = v(A)$.

If, following Priest, one subscribes to the T-Scheme but not to transparency, the following weaker conditions will suffice:

(Tr_1^*) $v(Tr(\ulcorner A \urcorner)) \geq v(A)$;
(Tr_2^*) $v(\ulcorner A \urcorner) > 0$ if $v(Tr(\ulcorner A \urcorner)) > 0$.

[7] The material conditional in LP is not detachable (counterexample provided in the main text below). Indeed, Beall et al. (2012) show that LP is entirely detachment-free, i.e. it cannot define a detachable connective. Detachment-free glut-theoretic approaches to semantic paradox are discussed in Goodship (1996), and have been recently revamped in Beall (2011).

[8] We make the simplifying assumption that every object on the domain serves as a name of itself. Conjunction, material implication and the universal quantifier are defined the standard way.

Then, the detachable conditional $\rightarrow$ may fail to satisfy contraposition, and, if A is a glut $Tr(\ulcorner A \urcorner))$ is allowed to be true only.[9]

We say that a sentence A is *satisfied* by a valuation v iff either $v(A) = 1$ or $v(A) = 0.5$. It is dissatisfied otherwise. Then, a sequent $\Gamma \vdash \Delta$ is *valid* iff no valuation satisfies all the sentences in Γ and dissatisfies all the sentences in Δ. LP is (famously) a *paraconsistent* logic: the principle of *ex contraditione quodlibet*

$$\text{ECQ}\ \frac{\Gamma \vdash A \qquad \Delta \vdash \neg A}{\Gamma, \Delta \vdash B},$$

is LP-invalid (just set $v(A) = 0.5$ and $v(B) = 0$). The Liar Paradox then becomes the proof of a *theorem*: where L is a Liar sentence equivalent to $\neg Tr(\ulcorner L \urcorner)$, the paradoxes establishes that both L and $\neg L$ are provable, and hence true.

2 ...and the Logic of Exclusive Denial

How to introduce *denial* in the foregoing framework? Following Priest (2006a, Sect. 6.1), we take assertion and denial to be external manifestations of, respectively, the mental states of acceptance and rejection. We represent them by means of *signed formulae*: $+A$ for assertion and $-A$ for denial, where $+$ and $-$ are non-embeddable force signs (Rumfitt 2000). Following Smiley (1996), we may interpret $+A$ and $-A$ as yes-or-no questions, respectively reading *A? Yes!* and *A? No!*.

Yes-or-no, or *bilateral*, axiomatisations of CL are well known (Smiley 1996; Rumfitt 2000; Humberstone 2000). They essentially comprise (i) what Rumfitt calls *signed positive analogues* of the standard rules for intuitionistic logic, i.e. standard rules such as $\vee$-I, written in a yes-or-no format

$$+\text{-}\vee\text{-I}\ \frac{\Gamma \vdash +A}{\Gamma \vdash +(A \vee B)},$$

(ii) rules for *denying* complex propositions, such as

$$-\text{-}\vee\text{-E}\ \frac{\Gamma \vdash -(A \vee B)}{\Gamma \vdash -A},$$

and (iii) what Rumfitt calls *coordination principles*: structural rules governing the logic of $+$ and $-$, such as the following:

$$\text{SR}\ \frac{\Gamma, +A \vdash +B \qquad \Delta, +A \vdash -B}{\Gamma, \Delta \vdash -A}.$$

One can give a *classical semantics* by defining a set of correctness-valuations C for signed formulae such that every member is induced by the set of admissible truth-valuations of CL by the following correctness clauses:

(C1) $v_c(+A) = 1$ iff $v(A) = 1$;
(C2) $v_c(-A) = 1$ iff $v(A) = 0$.

The assertion (denial) of A is correct just in case A is true (false). Validity for signed formulae is defined the obvious way: a sequent is valid iff it preserves value **1**, i.e. correctness.[10]

In presence of true contradictions, however, C1 needs revision. For according to glut theorists, one may correctly assert what's true, *even if* it turns out to be also false. That is, C1 must be modified as follows:

(C1*) $v_c(+A) = 1$ iff $v(A) \in \{1, 0.5\}$.

Symmetry considerations would suggest a corresponding revision of C2, that one may correctly deny what's false, *even if* it turns out to be also true:

(C2*) $v_c(-A) = 1$ iff $v(A) \in \{0, 0.5\}$.

But, if one could correctly deny what may be correctly asserted (a straightforward consequence of C1*, C2* and the existence of gluts), the denial of A would no longer be guaranteed to express disagreement. To see this, notice that, whatever the exact details of one's account of disagreement, two speakers disagree only if they have incompatible beliefs, or perform speech acts that cannot be jointly correct:

> *Disagreement*. Two speakers S_1 and S_2 who respectively utter u_1 and u_2 disagree only if u_1 and u_2 cannot both be correct.

This is why Lisa's assertion of (2) may not in general express disagreement with Marc's assertion of (1): in a glut-theoretic framework, utterances of (1) and (2) may both be true, and hence correct. To wit, let Q be glut, and suppose Marc asserts *it* but Lisa disagrees. If C2* held good, the assertion and the denial of Q would both be correct, and Lisa's denial of Q would not express disagreement. Thus, if denial is a means of expressing disagreement, then it must be *exclusive*: it must be 'impossible jointly to accept and reject the same thing' (Priest 2006b, p. 103). But, then, C2 must *not* be revised: the exclusive denial of A is correct iff A is (classically) *false only*, i.e. false but not true.

We thus get a *logic of exclusive denial*, call it LP*, whose language is the language of LP supplemented with

[9] More specifically, the set $\{Tr(\ulcorner L \urcorner) \rightarrow L, Tr(\ulcorner \neg L \urcorner), \neg L\}$ may fail to imply $\neg Tr(\ulcorner L \urcorner)$.

[10] Clauses C1 and C2 involve a a rather *objective* sense of correctness: the truth (falsity) of a proposition suffices for the correctness of its assertion (denial). The reader is free to interpret C1 and C2 evidentially, e.g. taking *evidence* for A's truth (untruth) to be sufficient for the correct assertion (denial) of A, and modify the logic accordingly (e.g. invalidate the Law of Excluded Middle and other classical principles).

signs expressing assertion (+) and denial (−). The semantics is given by a set of admissible correctness valuations V^*_{LP}, each member of which is induced by the admissible valuations of LP via C1* and C2. It validates many of the classical rules given in Smiley (1996) and Rumfitt (2000), but not all. For instance, the following two classically valid rules for negation

$$+\text{-}\neg\text{-E}\ \frac{\Gamma \vdash +(\neg A)}{\Gamma \vdash -A} \qquad -\text{-}\neg\text{-I}\ \frac{\Gamma \vdash +(A)}{\Gamma \vdash -(\neg A)}$$

are LP^*-invalid. The assertibility of $\neg A$ doesn't entail the deniability of A, for if A is both true and false, then $\neg A$ is correctly assertible, but A is not correctly deniable. Similarly, if the assertion of A is correct, it doesn't follow that the denial of $\neg A$ is also correct: if A is, once again, both true and false, the assertion of A is correct but its denial isn't.

On the other hand, the semantics validates the following, highly intuitive, coordination principles:

$$\mathsf{Coord}_1\ \frac{\Gamma, +A \vdash}{\Gamma \vdash -A} \qquad \mathsf{Coord}_2\ \frac{\Gamma \vdash +A \qquad \Delta \vdash -A}{\Gamma, \Delta \vdash}.$$

Coord_1 is a form of *reductio*: it effectively tells us that if A and all the members of Γ cannot be all correctly asserted, then asserting all the members of Γ will warrant the denial of A. Priest comes close to endorsing the principle in the following passage:

> An argument against an opponent who holds A to be true is rationally effective if it can be demonstrated that A entails something that ought rationally to be rejected B. For, it then follows that they ought to reject A. (Priest 2006a, p. 86)

As for Coord_2, it tells us that if Γ and Δ warrant the assertion *and* the denial of A, then one may not correctly assert all the members of Γ and Δ. That is, Coord_2 expresses the highly plausible principle, which leading glut theorists endorse, that one may not correctly assert and deny the same proposition. As Priest puts it, 'acceptance and rejection are mutually incompatible' (Priest 2006b, p. 103).

With Coord_1 and Coord_2 in place, though, trouble arises as soon as glut theorists try to *express* (exclusive) deniability.

3 Exclusive Denial and Revenge

Suppose the gap-theorist's language is rich enough to express *deniability*. That English contains some such predicate seems beyond doubt, as the following examples show:

(3) The judge is confident that everything Marc said is deniable.

(4) If what I say is deniable, why is nobody objecting?

Where T is a glut-theoretic extension of Peano Arithmetic (PA), with underlying logic LP^*, let us add to T's language a fresh predicate $\mathcal{D}(x)$ expressing correct deniability, i.e. such that $\mathcal{D}(\ulcorner A \urcorner)$ is true iff A is correctly deniable. Then, $\mathcal{D}(x)$ will satisfy the following:

(5) $v(\mathcal{D}(\ulcorner A \urcorner)) = 1$ iff $v_c(-A) = \mathbf{1}$.

It is easy to check that, given Coord_1 and Coord_2, the following rules governing the deniability predicate must then hold:

$$\mathcal{D}\text{-I}\ \frac{\Gamma, +A \vdash}{\Gamma \vdash +\mathcal{D}(\ulcorner A \urcorner)} \qquad \mathcal{D}\text{-E}\ \frac{\Gamma \vdash +A \qquad \Delta \vdash +\mathcal{D}(\ulcorner A \urcorner)}{\Gamma, \Delta \vdash}.$$

The rules have intrinsic intuitive appeal. The first says that, if A and all the members of Γ cannot be all correctly asserted, then asserting all the members of Γ will warrant the assertion that A is deniable. The second tells us that one may not correctly assert both that A and that A is deniable. With these rules in place, it is now a routine exercise to show that T is incoherent, i.e. that it licenses both the assertion and the denial of the same sentence.

3.1 The Paradox of Deniability

This is shown by a variant of Littman's Irrationality Paradox (Littman 1992), viz. an argument involving a sentence which says of itself (only) that it is rationally deniable. Let D be such a sentence. Then, D satisfies:

$$\frac{+D}{+\mathcal{D}(\ulcorner D \urcorner)} \qquad \frac{+\mathcal{D}(\ulcorner D \urcorner)}{+D}$$

One may then reason thus:

$$\mathcal{D}\text{-I}\ \cfrac{\text{Contraction}\ \cfrac{\mathsf{Coord}_2\ \cfrac{(5),\text{C2}\ \cfrac{\cfrac{+D \vdash +D}{+D \vdash +\mathcal{D}(\ulcorner D \urcorner)}}{+D \vdash -D} \qquad +D \vdash +D}{+D, +D \vdash}}{+D \vdash}}{\vdash +\mathcal{D}(\ulcorner D \urcorner)}$$

By definition of D, $+D$ immediately follows. But, by (5) and C1 (right-to-left), $-D$ also follows. Thus, D is both assertible and deniable. Similar paradoxes can be generated for any predicate Θ satisfying Θ-analogues of $\mathcal{D}$-I and $\mathcal{D}$-E, such as e.g. 'is trivial', 'is rejectable', 'is not rationally acceptable' etc. Call this the *Paradox of Deniability*.

To see why the foregoing reasoning is a paradox, it is enough to reflect on the fact that the assumptions required to 'prove' $+D$ and $-D$ are quite minimal: the standardly accepted structural rules, some means of generating self-reference, the claim that denial is exclusive, as codified by C2 (semantically) and Coord_2 (syntactically), and (5), viz. the claim that the deniability predicate expresses exclusive denial. Assuming that self-reference isn't the culprit, glut theorists who accept the standard structural rules are left but with two uncomfortable options: either deny that denial is exclusive, or deny that exclusive denial is expressible.[11] Notice also that, in a LP^* setting, the Liar Paradox doesn't entail that the Liar sentence L is both assertible and deniable. In LP^*, one can prove both $+L$ and $+(\neg L)$. But, since $v(L) = 0.5$, i.e. L is false but also true, we are not thereby licensed to deny L. By contrast, here the assumption that $\mathcal{D}(x)$ expresses exclusive denial validates the inference from $+\mathcal{D}(\ulcorner D \urcorner)$ to $-D$, and thus allows us to conclude that D is both assertible and deniable.

★★ *Parenthetical note*. Hartry Field informally considers a paradox of acceptance and rejection, resting on analogues of $\mathcal{D}$-I and $\mathcal{D}$-E (Field 2008, fn. 8, p. 95). However, he professes not to be worried by such paradoxes, on the grounds that one can always drop the Law of Excluded Middle (LEM) for acceptability and deniability claims:

> I don't know that there is any strong reason to worry about ... paradoxes involving normativity: there tend to be more resources for dealing with apparent normative paradoxes than there are for the paradoxes of truth alone ...we could reject excluded middle. (Field 2008, pp. 77–78)

However, Field fails to notice that LEM isn't essentially used in the deniability paradoxes—a point already made in Priest (2010a, Sect. 7), who also considers a variant of the Irrationalist Paradox (see also Priest 2006a, pp. 111–112). Where $\mathcal{R}$ is a predicate expressing *rational acceptance* and R is a sentence equivalent to $\neg\mathcal{R}(R)$, Priest (2010a, p. 121) shows, assuming (i) the validity of the schema

$$\neg\mathcal{R}(A \wedge \neg\mathcal{R}(A))$$

and (ii) that rational acceptance is closed under single-premise deducibility, but without assuming LEM, that R is both rationally acceptable and not rationally acceptable. Priest concludes that while 'a dialetheist can accept this', Field, who wishes to preserve consistency, can't (Priest 2010a, p. 121). Here we simply add that, if the foregoing considerations are correct, the Paradox of Deniability is a problem for *Priest* too, since it isn't simply a 'proof' of $A \wedge \neg A$: it also 'proves' that a certain sentence is both assertible and deniable. *End note*. ★★

[11] Glut theorists may avoid the problem by restricting some of the standardly accepted structural rules, such as $\mathsf{Contraction}$ (that if $\Gamma, A, A \vdash B$, then $\Gamma, A \vdash B$). For reasons of space, however, we cannot consider this option here. For some recent substructural approaches to the semantic paradoxes, (see e.g. Shapiro 2011; Zardini 2011; Ripley 2011b). For a recent attempt to create a revenge problem for the substructural theory presented in Zardini (2011), see Murzi (2014).

3.2 Norms of Denial

The Paradox of Deniability shows that glut theorists must deny either that denial is exclusive, and consequently drop Coord_2 and substitute C2 with C2*, or that exclusive denial is expressible. It seems to us that deniability *is* expressible in the language, as examples (3) and (4) show. If that's correct, the real issue is whether denial in a glut-theoretic framework can be exclusive. Let us assume that it is. Then, one obvious challenge arises as soon as one tries to express in the object language norms for exclusive denial such as the one codified by C2. The norm asserts that A is correctly deniable iff A is *false only*. But obviously such a norm, in its intended interpretation, cannot be expressed in the glut theorist's language: falsity only just is Boolean negation—a notion that, on the foregoing assumptions *must* be deemed incoherent once Tr-I and Tr-E are in place (see e.g. Beall 2007, pp. 5–6). Should one conclude that denial isn't exclusive after all?

Perhaps for this reason, Priest sometimes questions the exclusivity of denial. He considers two norms of denial, neither of which makes denial exclusive. We think, however, that both norms are problematic. We consider them in turn.

The first norm takes *untruth* to be the aim of denial:

Deny(U) You may deny A if there is good evidence for A's untruth.

However, we doubt that this the norm glut theorists really need. The reason: in presence of (i) a plausible corresponding norm for assertion

Assert(T) You may assert A if there is good evidence for A's truth,

and (ii) the existence of gluts such as the Liar sentence, **Deny(U)** licenses us to accept and reject the same sentence.

Priest doesn't think this is a problem. He takes the joint assertibility and deniability of a given sentence to be a *rational dilemma*, on the further assumption that it is metaphysically 'impossible to accept and reject the same thing' (Priest 2006a, p. 86). As he puts it:

> [b]y these two principles, one ought to accept A and one ought to reject A. We have a dilemma, since we cannot do both. (Priest 2006a, pp. 110–111)

Rationality demands that we do something that cannot be done but, Priest adds, 'arguably, the existence of dilemmas is a fact of life' (Priest 2006a, pp. 111). We agree that dilemmas are facts of life, but we don't think that the appeal to rational dilemmas can help in the present case.

To begin with, one might take exception with Priest's assumption that it is metaphysically impossible to both assert and deny the same proposition. For instance, it seems to us that the following is a case in which the same proposition is jointly asserted and denied:

Marc: Is Dover North of London?

Lisa: Yes! And no!

Marc: ??

Glut theorists might insist that Lisa here has changed her mind, and thus has not both asserted and denied that Dover is North of London; she has rather *first* asserted and *then* denied that content. However, we don't think this would be a good description of the case at hand: there seems to be something wrong with Lisa's answer, which isn't reducible to a sudden change of heart. For instance, if asked whether she has changed her mind, and settled on the view that Dover isn't North of London after all, Lisa might insist that she hasn't, and that she is indeed prepared to both assert and reject the claim that Dover is North of London. By our lights, Lisa's behaviour would be most plausibly interpreted as evidence that she is in effect willing to both assert and to deny that Dover is North of London.

Be that as it may, there are good reasons for thinking that Priest's claim that 'acceptance and rejection are mutually incompatible' (Priest 2006a, p. 86) should be interpreted as saying that one *shouldn't* both assert and deny the same sentence at the same time. The main problem with **Deny(U)**, in its non-classical interpretation, is that the norm effectively licenses the denial of propositions (such as the proposition expressed by the Liar sentence) that, by the glut theorist's lights, we ought to assert, thus making denial non-exclusive, *and hence unable to express disagreement*. For instance, given **Deny(U)**, Lisa's denial of (1) would be no longer guaranteed to express disagreement, since the assertion and denial of (1) could then *both* be correct. If denial is to serve as a means to express disagreement, it must be *rationally impermissible* to both assert and deny A. Yet, in view of the Paradox of Deniability, no comprehensive set of norms for exclusive denial can be formulated in the glut-theorist's language. Such a set would have to register the fact that correct denial 'aims' at falsity only, just like assertion aims at truth (Dummett 1959). But this would require the norms to involve notions that cannot be expressed in the foregoing glut-theoretic framework, on pain of incoherence. In our view, this makes denial in LP^* somewhat ineffable: in absence of an appropriate norm, it is hard to see, for instance, how incorrect denials can be criticised. One can deem denials as correct, or incorrect. However, one cannot say *why* a given denial was correct.

We take the fact that Priest proposes an second norm for denial which is arguably aimed at avoiding the present difficulty as evidence that Priest himself may feel the force of the considerations we have just rehearsed. The norm is this:

Deny(U)* You may deny A if there is good evidence for A's untruth, *unless there is also good evidence for its truth.*

In short: one may deny A if one has good reasons for thinking that A is untrue *only*. This second norm, though, makes denial profoundly *unlike* assertion. Unlike assertion, any denial may later turn out to be incorrect, since any false sentence can in principle be discovered to be a glut . Thus, you can disagree with my assertion that $0 \neq 0$, and thus deny $0 \neq 0$. But, even if you can prove $0 = 0$, and hence disprove $0 \neq 0$, you can never be fully confident that your denial is correct: a proof of $0 \neq 0$ may always turn up. By contrast, if you have proved $0 = 0$ and thereby assert it, you can be fully confident that your assertion is correct. We find this asymmetry problematic: nothing in our practice of asserting and denying things, it seems to us, suggests that assertion can be indefeasible in a way that denial is not.

In sum, then: exclusive semantic notions are needed to express norms governing exclusive denial. In turn, exclusive denial would appear to be needed, in a glut-theoretic framework, in order to express disagreement. But if no norms for exclusive denial can be non-trivially formulated in such a framework, the claim that exclusive denial can serve as a means to express disagreement would appear to lose much of its initial appeal. Or so we have argued.

3.3 Objections and Replies

We now turn to briefly consider, and address, some potential objections to our claim that the Paradox of Deniability is a genuine semantic paradox showing that glut theorists cannot express exclusive denial.

Following Parsons (1984, Sect. 3) and Priest (2006a, Sect. 6.4), it may be objected that there are no paradoxes of denial and that, for this reason, the argument of Sect. 3.1 purporting to show that deniability is not expressible must fail. Thus, Priest writes:

> attempts to formulate distinctive Liar paradoxes in the form of denial fail, since [$-$], being a force operator, has no interaction with the content of what is uttered. (Priest 2006a, p. 108)

However, this objection is beside the point in the present context: the deniability *predicate* is not a force operator.

A more telling objection would be to argue that $\mathcal{D}$-E must be invalid, on the grounds that there are valuations $v \in V_{LP}$ such that $v(A) = v(\mathcal{D}(\ulcorner A \urcorner)) = 1$. The assertion of both A and $\mathcal{D}(\ulcorner A \urcorner)$ would then be correct, and the rule would not preserve correctness.

The problem with this, though, is that the valuations in question are ruled out by the principle that we may not correctly deny what's true. More precisely, that no such valuation is admissible is a consequence of C2, that the denial of A is correct iff A is false only, together with (5), the principle that $v(\mathcal{D}(\ulcorner A \urcorner)) = 1$ iff the denial of A is correct. Rejecting either principle would seem problematic, however. On one hand, C2 codifies the exclusivity of denial: it guarantees that, A is in effect deniable iff A is false only. But as we have seen in Sects. 1 and 3.2, denial *must* be exclusive, if it is to serve as a means to express disagreement. On the other, (5) ensures that the deniability predicate expresses such an exclusive notion of denial: it guarantees that the claim is deniable is true iff A is in effect deniable.

Glut-theorists might still insist that 'there is nothing [one] can assert that entails disagreement' (Priest 2006b, p. 292) and that, for this reason, the assertion that A is deniable need not imply the denial of A. That is, they may hold on to C1* and C2, but reject the left-to-right direction of (5): one may correctly assert that A is deniable even if A isn't deniable, and hence false only. But this, again, would be to give up the assumption that $D(x)$ expresses exclusive deniability.

★★ *Parenthetical note.* Priest effectively concedes that in 'normal conditions' the denial of A may be expressed by asserting $A \rightarrow \bot$, where $\rightarrow$ is a detachable conditional (Priest 2006a, p. 105) and $\bot$ is a 'a logical constant such that it is a logical truth that $\bot \rightarrow A$, for every A' (Priest 2006a, p. 85).[12] It is just, Priest adds, that this isn't always guaranteed to work: trivialists accept $\bot$, and hence can legitimately assert *both* A and $A \rightarrow \bot$. As Priest puts it:

> In most contexts, an assertion of [...] $A \rightarrow \bot$ would constitute an act of denial. Assuming that the person is normal, they will reject $\bot$, and so, by implication, A. The qualifier 'in most contexts' is there because if one were ever to come across a trivialist who accepts $\bot$, this would not be the case. For such a person an assertion of $[A \rightarrow \bot]$ would not constitute a denial: nothing would. (Priest 2006a, pp. 105–106)

Simply put, then, Priest's idea is this: if $\bot$ entails triviality, then *in most contexts* an assertion of $A \rightarrow \bot$ will be equivalent to the exclusive denial of A. For, in most contexts, there will not be trivialists—speakers who accept everything, including $\bot$. Then, since $\rightarrow$ detaches, an assertion of A in presence of $A \rightarrow \bot$ would entail triviality. Thus, if we interpret $\mathcal{D}(\ulcorner A \urcorner)$ as $A \rightarrow \bot$, in most contexts the left-to-right direction of (5) holds: $\mathcal{D}(\ulcorner A \urcorner)$ is true only if A is false only, as the Paradox of Deniability assumes. However, Priest observes, in *some* contexts there will be trivialists. Hence, he concludes, the assertion of $A \rightarrow \bot$ cannot in general be equivalent to the exclusive denial of A, since trivialists *accept* $\bot$, and (5) must fail.

We limit ourselves to making two simple observations. First, if following Burge (1979, 1986) and others, we assume a modicum of *semantic externalism*, i.e. that the content of an expression is in general fixed (at least in part) by social factors external to the speaker, then it seems to us that, if $\rightarrow$ detaches and $\bot$ entails the conjunction of the sentences of the language, then then set $\{A \rightarrow \bot, A\}$ entails triviality irrespective of what a trivialist might think. Second, on a more internalist view of language, notice that Priest isn't a trivialist. Hence, at least in his idiolect, he will be able to deny A, in the sense of excluding A's falsity, by asserting $A \rightarrow \bot$. More generally, glut theorists who reject trivialism and accept that $\rightarrow$ detaches, *must* reject A, if they accept $A \rightarrow \bot$. Thus, if the glut theorist interprets $\mathcal{D}(\ulcorner A \urcorner)$ as being equivalent to $A \rightarrow \bot$, then $\mathcal{D}$-E, and hence the Paradox of Deniability, would be validated *for her*.[13] *End parenthetical.* ★★

4 Shrieking Gluts and Disagreement

If one's theory is not trivial, one can ensure that A is false only by *denying* A. But, we've argued in Sect. 3.2, no adequate norm of exclusive denial can be stated in the glut theorist's language; more specifically: one cannot express that A's denial is correct only if A is *false only*. A different tack might do, however. Beall has recently suggested the following recipe for ensuring, up to triviality, that A is either true or false only: one simply adds to one's theory a rule of the form $A \wedge \neg A \vdash \bot$.[14] Then, one is either committed to A's truth (falsity) only, or to trivialism. In Beall's view, such rules—*shriek* rules, as he calls them—hold the key for solving the 'just true (false)' problem—as he puts it, 'the challenge to glut theorists: how can you (viz., glut theorists) enjoy this common notion of a just true [false] theory ... (Beall 2013, p. 440). Or do they?

12 Accordingly, the obvious interpretation of $\bot$ is as follows:

$(\bot)\ v(\bot) = 0.$

13 We'll return to this point in Sect. 4 below; see especially Sect. 4.1 and fn. 16.

14 We note in passing that the role of shriek rules is similar to that of the coordination principle Coord_2 in LP^*. The latter ensures that A may not be coherently both asserted and denied, on pain of triviality.

4.1 Shrieking Gluts

Beall's starting point is Priest's suggestion that, in most contexts, one may deny A, and hence ensure that A is false only, by asserting $A \rightarrow \bot$ (Priest 2006a, pp. 105–106; see also the parenthetical note in Sect. 3.3 above).[15] His next step is the observation—also made by Priest—that A's denial, so conceived, ensures the following: that either A is false only, or trivialism is true. Thus, the assertion of $A \rightarrow \bot$ is equivalent to exclusive denial *up to triviality*. But this, both Beall and Priest observe, is exactly what happens in a classical framework. Here, too, we have two possibilities: either A is true or false only, or, if there is overlap, trivialism follows via ECQ. It would seem that that, for this reason, both the glut theorist and the classical logician can effectively 'enjoy th[e] common notion of a just true [false] theory' (Beall 2013, p. 440).

Beall notices one problem with the proposal as it stands, however. According to Priest, $A \rightarrow B$ expresses a 'logical connection' between A and B. But, someone (a relevant logician) might argue, there is no logical connection between A and $\bot$: the fact that A is false only should not *logically* entail triviality. Hence, $A \rightarrow \bot$ is 'much too strong', and $A \rightarrow \bot$ is 'virtually never true' (Beall 2013, p. 441).[16,17] Beall's suggested way out is to add, for any 'consistent', i.e. non-glutty A, a local, and hence *non-logical*, shriek rule of the form $A \wedge \neg A \vdash \bot$. This suffices to ensure, up to triviality, that A is either true or false only: given $A \wedge \neg A \vdash \bot$, if $\vdash A$ $(\vdash \neg A)$, then $\vdash \neg A$ $(\vdash A)$ only if $\vdash$ is trivial. More generally, in order to ensure that a predicate P_n does not deliver gluts, i.e. true sentences of the form $P(a_1, \ldots, a_n)$ and $\neg P(a_1, \ldots, a_n)$, Beall suggests that one adds to one's system the following shriek rule for P_n:

$$\exists x_1, \ldots, \exists x_n (Px_1, \ldots, x_n \wedge \neg Px_1, \ldots, x_n) \vdash_T \bot$$

Up to triviality, this guarantees that sentences of the form $P(a_1, \ldots, a_n)$, and truth-functional compounds thereof, are either true or false only. Problem solved, or so Beall argues.[18]

Shriek rules naturally suggest a two-step solution to the problem of disagreement. If Lisa disagrees with Marc's assertion that (1), she may first add the non-logical rule

$$\frac{\text{Dover is North of London and Dover is not North of London}}{\bot}$$

to her system, and then express her disagreement, exactly as the classical logician would, by asserting (2), viz. that Dover is not North of London. In presence of the rule, her assertion guarantees that Marc's assertion of (1) yields triviality. Beall's shrieking proposal therefore suggests the following account of exclusive denial—in effect, a version of *Classical denial*:

Deny(S) You may deny A if (and only if) $\neg A$ is assertible (there is good evidence for $\neg A$'s truth) and $A \wedge \neg A \vdash \bot$.

More simply:

Deny(S)* You may deny A if $A \vdash \bot$.

Will this do?

4.2 Two Objections

There are two main problems with the foregoing proposal, or so we'd like to suggest.

To begin, the classical logician will object that there is a clear sense in which the problem of expressing truth and falsity only, and related notions such as exclusive deniability, has not yet been solved. Beall asks how glut theorists can 'enjoy this common notion of a just true theory (or just-false theory ...)' (Beall 2013, p. 440). However, the notions mentioned here are decidedly not in common between the classical logician and the glut theorist. In

[15] Similarly, asserting $A \wedge (\neg A \rightarrow \bot)$ ensures, up to triviality, that it is true only.

[16] There is a second problem. Consider a Curry sentence C such that $Tr(\ulcorner C \urcorner) \leftrightarrow (C \rightarrow \bot)$, where $Tr(x)$ satisfies at least Tr-I and Tr-E. If C were true then $C \rightarrow \bot$ would also be true; but then, by *modus ponens* (which holds for Priest's conditional), it would be possible to derive $\bot$. The glut theorist is thus forced, on pain of trivialism, to reject C. If C is rejected, however, $C \rightarrow \bot$ should also be rejected. But then $A \rightarrow \bot$ cannot be used to express in general the rejection of A: in order to express the rejection of A, $A \rightarrow \bot$ must be true (and correctly denied sentences must be false only). As far as we can see, this effectively undermines Priest's proposal.

[17] It might be thought that a similar objection might be levelled against Coord$_2$ within the LP* framework we introduced in Sect. 2. That is, one might argue that, contrary to what Coord$_2$ effectively states, there shouldn't be a *logical* connection between the joint assertion and denial of A and $\bot$. However, it is not obvious that the objection would apply: unlike Priest's entailment connective, which is intended to express logical consequence, LP*'s consequence relation need not be logical.

[18] The Curry problem we mentioned in footnote 16 above doesn't obviously arise in the present context, for at least two reasons. First, in order to replicate the relevant Curry-like reasoning, one would have to define a sentence Q in some sense equivalent to $Q \vdash \bot$. But, it would seem, such a sentence would be bound to be ungrammatical. To be sure, one could define a sentence Q equivalent to a sentence to the effect that the argument $\langle Q \therefore \bot \rangle$ is valid. This would give rise to a validity-involving version of Curry's Paradox, the v-Curry Paradox (Beall and Murzi 2013). However, Beall rejects one of the key semantic ingredients of the paradox, a rule to the effect that A and the the claim that the argument $\langle A \therefore B \rangle$ is valid jointly entail B. This would effectively block the relevant Curry-like reasoning. Second, and relatedly, Beall's logic LP doesn't enjoy a detachable conditional (see fn. 7 above), which, again, suffices to block the Curry-like reasoning sketched in fn. 16.

classical theories, one can express *in the object language* that A is false only: an assertion of $\neg A$ will do (up to triviality) just that. Indeed, in the best classical theories, an assertion of '$\neg A$ is true' will also do (see e.g. Parsons 1974; Glanzberg 2001).[19] By contrast, this possibility would appear to be foreclosed to the gap theorist. Asserting $\neg A$ won't do, since A may be a glut. And a straightforward variant of the Liar paradox involving a Liar sentence 'L is false only' shows that 'A is false only' won't do either. The paradox shows, following standard Liar reasoning, that L is both true and false only. Thus, 'true only' and 'false only' do not have disjoint extensions in the glut theorist's language, and hence must lack their intended interpretation.[20] While glut theorists can certainly *say* that 'a closed theory is just true', they will thereby 'succeed in ruling out gluttiness up to triviality ...[i.e.] imply that it's either (negation-) consistent or that it's trivial' (p. 445) *only if, in their mouth, 'just true' has its intended interpretation*. But this cannot be.

The second difficulty is this. Shrieking rules allow glut theorists to recapture classical theories in non-classical theories with underlying logic LP. One simply needs to *fully shriek* the theory in question: that is, add shriek rules for every axiom of the theory (Beall 2014, Theorem 2). However, this raises the question which theories should be shrieked, and why. Pending an adequate answer, the shrieking proposal is empty: if we are not told *why* one may not add shrieking rules to inconsistent theories, nothing would prevent one from doing so, with catastrophic effects. Beall offers two different answers to the question when to fully shriek a theory and why: a metaphysical one, and a more deflationary one. We consider them in turn.

The starting point of the metaphysical answer is an assumption about the nature of the domain described by a given theory. Here are two representative quotes:

> What motivates the shriek rules is the rejection that the given sentences are anything but, in a phrase, 'just true'. (Beall 2014, p. 443)

> Let us suppose that we take a domain (or phenomenon) to be consistent; we take its true theory to be the sort of theory for which 'classical recapture' makes sense. ...In taking the given domain to be consistent, we reject that the true axiomatic theory is inconsistent; we reject that there are predicates of the theory's language that deliver gluts. (Beall 2013, Sect. 4)

In short: one is justified in adding shrieking rules to a theory T when one takes the theorems of T to be *true only*. Then, once fully shrieked, T is either 'consistent', in the sense that its sentences are either true or false only, or 'trivial' (Beall 2013, p. 444).

However, the intended sense of consistency, viz., non-gluttiness, cannot be expressed in the glut theorist's language. Just like $\neg A$ does not express falsity only since A could be a glut, $\neg(A \wedge \neg A)$ does not express that A has exclusive semantic status, i.e. that A is either true or false only, since $\neg(A \wedge \neg A)$ is already a theorem of LP, and hence holds for *all* sentences, including the glutty ones. A stronger sense of consistency is needed: one implying that A is either true or false *only*. But, as we've seen, no such notion can be expressed in the glut theorist's language.

Perhaps for this reason, Beall's *official* account of what to shriek when is more deflationist in spirit, and doesn't appeal to semantic notions (Beall 2014, Sect. 5). It is, essentially, the epistemic conservatism defended by, among others, Gil Harman (1986). The idea is that we are justified in accepting a classical theory such as PA simply because its theorems are part of our initial stock of beliefs, and we have no good reasons to doubt their truth. Beall writes:

> we are (at least prima facie) justified in maintaining what we accept and reject until we have some special reason to change. And with the vast majority of thinkers, I see no good reason to accept that the non-semantic realm (arithmetic, physics, etc.) might be hiding some metaphysical 'glutty' (contradictory) nature. (Beall 2014, Sect. 5)

We find this problematic too, however. While we agree that explanations must come to an end somewhere, we don't think they should come to an end *too soon*, on pain of being too weak. Consider, for instance, the case of set-theory. According to some (Priest 2006a, b), Russell's paradox is a 'special reason' to revise our initial belief that set-theory is consistent. However, the majority of set-theorists, following the likes of Zermelo and, more recently, Boolos, have since defended a consistent conception of the universe of sets—one according to which such a universe is built in stages, known as the *iterative* conception (Boolos 1971). Arguably, on the iterative conception of set, one's grasp of the universe of sets (the cumulative hierarchy of ever larger sets) *itself* grounds the belief that set-theories such as ZFC are consistent. Similarly, it might be suggested that one's grasp of the natural number structure grounds the belief that PA is consistent. PA is consistent *because* the PA

[19] To be sure, unlike glut theorists, classical logicians may not express *naïve truth*, i.e. a notion of truth satisfying (at least) *Tr*-I and *Tr*-E. But this is how it should be: classical logicians and glut theorists negotiate the trade between consistency (non-triviality) and expressibility at different points.

[20] Beall (2009) argues that truth and truth only really are the same notion, so that both can overlap with falsity and falsity only. But I take the present shrieking proposal to be a change of heart.

axioms satisfy the natural number structure. However, neither explanation would be available to someone whose *only* admissible grounds for holding that theories such as PA and ZFC are consistent is that most people think so, and that we haven't found a contradiction yet in such theories.

We conclude that Beall's shrieking proposal essentially faces the same difficulties that, as we have argued in Sect. 3, LP^*. In both frameworks, glut theorists can express disagreement with the assertion that *A*. They can do so by *denying A*, in an exclusive sense of denial. However, they may not express exclusive denial, and kindred notions such as falsity only, in their object language. This makes it hard for them to formulate *norms* for denial, and hence threatens to make the notion of denial ineffable.

5 Concluding Remarks

Denial plays a key role in the standard glut-theoretic account of disagreement: glut theorists may express disagreement with someone's assertion that *A* by *denying A*, or so the account goes. In order for this to work, denial must be exclusive: one may not correctly assert and deny the same proposition. Then, LP can be expanded into a bilateral logic of exclusive denial, LP^*. However, as shown by the Paradox of Deniability, *exclusive deniability*, a key semantic notion of the logic, is not expressible in the glut theorist's language. Beall's suggestion that shrieking rules allow glut theorists to 'enjoy' exclusive notions such as falsity only, and indeed exclusive deniability, won't solve the problem either, or so we have argued. Shrieking rules allow glut theorists to *ensure* that a sentence *A* is false only. But, as before, the notion of falsity (truth) only cannot be *expressed* in the glut theorist's language. Hence, glut theorists may not use it to *justify* their choice to shriek one theory rather than another; and, in particular, to justify their choice to deny one sentence rather than another. We conclude that glut theorists are faced with a dilemma: either denial can serve as means to express disagreement, but the notion of exclusive deniability isn't expressible in the glut theorist's language, or deniability is expressible, but denial may no longer serve as a means to express disagreement.

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